



дец 10/20

**МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
имени М.В.ЛОМОНОСОВА**

Вариант 3

Место проведения Москва
город

*Выход: 14:26
Вход: 14:28*

*+1 мес
+1 мес
+1 мес*

ПИСЬМЕННАЯ РАБОТА

Олимпиада школьников Ломоносов
наименование олимпиады

по математике
профиль олимпиады

Гришина Диана Сергеевна
фамилия, имя, отчество участника (в родительном падеже)

Шифр	Сумма	1	2	3	4	5	6	7	8
81-88-57-82	70	15	15	15	0	15	10	0	0

№1

Числовит

$$\left[\sqrt{45 + \sqrt{2013}} - \sqrt{45 - \sqrt{2013}} \right] =$$

$$= \left[\sqrt{\frac{45 + \sqrt{2}}{2}} - \sqrt{\frac{45 - \sqrt{2}}{2}} \right]$$

$$= \left[\sqrt{\left(\sqrt{\frac{45 + \sqrt{2}}{2}} + \sqrt{\frac{45 - \sqrt{2}}{2}} \right)^2} - \sqrt{\left(\sqrt{\frac{45 + \sqrt{2}}{2}} - \sqrt{\frac{45 - \sqrt{2}}{2}} \right)^2} \right]$$

находим.

$$\left(\sqrt{\frac{45 + \sqrt{2}}{2}} + \sqrt{\frac{45 - \sqrt{2}}{2}} \right)^2 = \frac{45 + \sqrt{2}}{2} + \frac{45 - \sqrt{2}}{2} + \sqrt{45^2 - 2} =$$

$$= 45 + \sqrt{2013} \text{ в точной форме.}$$

$$= \left[\sqrt{\frac{45 + \sqrt{2}}{2}} + \sqrt{\frac{45 - \sqrt{2}}{2}} - \sqrt{\frac{45 + \sqrt{2}}{2}} + \sqrt{\frac{45 - \sqrt{2}}{2}} \right] =$$

$$= \left[\sqrt{90 - 2\sqrt{2}} \right] = \left[\sqrt{\left(\sqrt{45 + \sqrt{2013}} - \sqrt{45 - \sqrt{2013}} \right)^2} \right]$$

$$45 + 45 - 2\sqrt{45^2 - 2} = 90 - 2\sqrt{2}$$

$$\left[\sqrt{45 + \sqrt{2013}} \right]$$

$$\sqrt{90 - 2\sqrt{2}} < \sqrt{90 - 2\sqrt{2}} < \sqrt{90} < \sqrt{100} = 10.$$

$$\frac{11}{\sqrt{87}}$$

$$\frac{11}{\sqrt{87}}$$

$$\Rightarrow 9 < \sqrt{90 - 2\sqrt{2}} < 10.$$

⇓

$$\left[\sqrt{90 - 2\sqrt{2}} \right] = 9$$

Ответ: 9.

Числовый

$|\alpha| \rightarrow \min.$

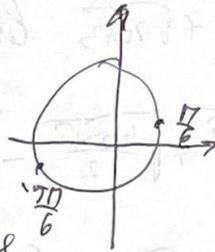
Функция $[-\pi; \pi]$.

$$|234 \sin^{20}(x + \frac{\pi}{3}) - 789 \cos^{23}(2x + \frac{\pi}{3})| = 2023$$

$$\begin{aligned} |234 \sin^{20}(x + \frac{\pi}{3})| \leq 234 \\ -789 \cos^{23}(2x + \frac{\pi}{3}) \leq 789 \end{aligned} \Rightarrow \text{так как } |234 \sin^{20}(x + \frac{\pi}{3}) - 789 \cos^{23}(2x + \frac{\pi}{3})| \leq |234 + 789| = 2023$$

$$\begin{cases} \sin^{20}(x + \frac{\pi}{3}) = 1 \\ \cos^{23}(2x + \frac{\pi}{3}) = -1 \end{cases} \Rightarrow \begin{cases} \sin(x + \frac{\pi}{3}) = \pm 1 \\ \cos(2x + \frac{\pi}{3}) = -1 \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{\pi}{6} + \pi k \quad (1) \\ 2x + \frac{\pi}{3} = \pi + \pi t \end{cases}$$



(1) на $[-\pi; \pi]$ даёт два решения

$$\begin{cases} x = \frac{\pi}{6} \\ x = -\frac{\pi}{6} \end{cases}$$

тогда нужны такие d : $789 \in \mathbb{Z}$:

$$\begin{cases} \frac{\pi d}{6} + \frac{\pi}{3} = \pi + \pi k \\ -\frac{2\pi d}{6} + \frac{\pi}{3} = \pi + \pi t \end{cases}$$

$$\frac{d}{6} + \frac{1}{3} = 1 + \pi t$$

$$d = 4 + 12t \Rightarrow d = \dots, -8, 4, 16, \dots$$

$$-\frac{2d}{6} + \frac{1}{3} = 1 + \pi t \Rightarrow -5d = 4 + 12t \Rightarrow d = -\frac{4}{5} - \frac{12t}{5} \Rightarrow \frac{4+12t}{-5} = -\frac{4}{5} - \frac{12t}{5}$$

$$\Rightarrow d = 4, \frac{8}{5}; -\frac{4}{5}; -\frac{16}{5}$$

Наименьшее по модулю $d \neq 0$ $d = -\frac{4}{5}$.

Ответ: $d = -\frac{4}{5}$. $k = \frac{4}{5}$.

81-88-57-82
(89.13)

$a_1 = 13$ Чирков $\begin{matrix} 1234 \\ 789 \\ 2022 \end{matrix}$ a_{2022} $\frac{a_1 + \dots + a_{2022}}{2022}$

$$\frac{a_n}{a_{n-1}} = \frac{(n^2+1) \cdot n}{(n-1)^2+1}$$

Куцаев, $n \leq$

$$a_{n+1} = k \cdot (a_1 + \dots + a_n)$$

$$a_n = \frac{(n^2+1)n!}{2} a_1$$

$90 - 2\sqrt{2} = a^2 + b$
 $90 = a^2 + b^2$
 $2\sqrt{2} = ab$

$$a_{n+1} = a_n \cdot \frac{(n^2+1)n!}{(n-1)^2+1} = 90 = \frac{a^2 b}{2a + a^2}$$

$$90 = \frac{2}{a^2} + a^2$$

$$\sum_{k=1}^{2022} \frac{(k^2+1)k!}{2} a_1 = \frac{(2022^2+1)2022!}{\sum_{k=1}^{2022} ((k^2+1)k!)}$$

$a^4 - 90a^2 + 2 = 0$
 $a_1 = 4 \pm \sqrt{16-2} = 2023$

$$\frac{a_n}{a_{n-k}} = \frac{(n^2+1)n!}{((n-k)^2+1)(n-k)!}$$

$a^2 = 4 \pm \sqrt{16-2}$
 $a \sqrt{2} = 1, 11$
 $k, p, 2$

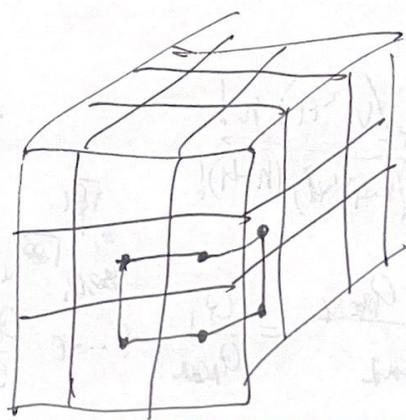
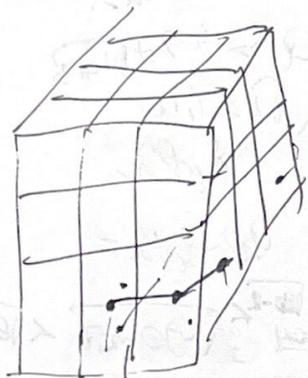
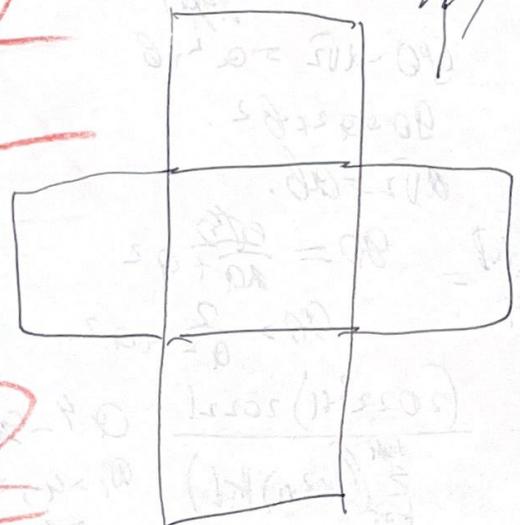
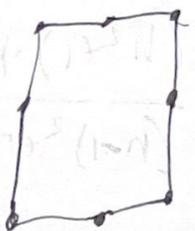
$$\frac{1}{k} = \frac{a_1 + \dots + a_{2022}}{a_{2022}} = \frac{a_1}{a_{2022}} + \dots + \frac{a_{2021}}{a_{2022}} < \sqrt{90-2\sqrt{2}} < \sqrt{90} < 10$$

$$\sum \frac{2}{(2022^2+1) \cdot 2022!} + \frac{2k}{2! \cdot 2022!} \dots \sum_{k=1}^{2022} \frac{a_k}{a_{2022}} >$$

$$= \sum_{k=1}^{2022} \frac{(k^2+1)k!}{(2022^2+1) \cdot (2022-k)!}$$

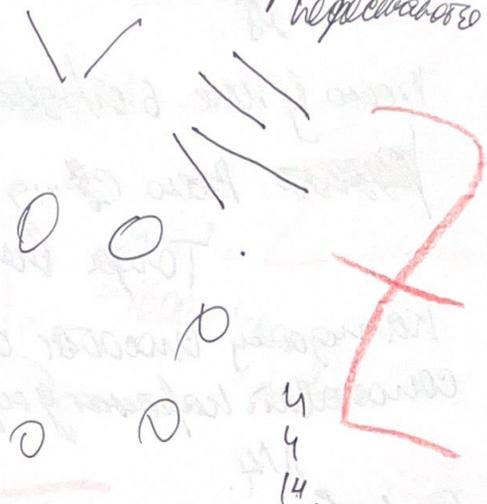
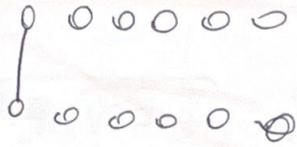
$$\frac{Q_n}{Q_{n-1}} = \frac{(n^2+1) \cdot n}{(n-1)^2+1}$$

Упрощен



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(89,13)

Через нее внах не
вот селомого



$$\frac{1}{4} \sqrt{(a+b+c-d)(a+b-c+d)(a-b+c+d)(-a+b+c+d)}$$

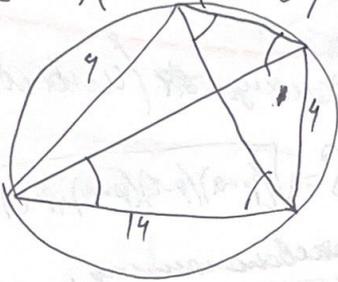
$$= \frac{1}{4} \sqrt{(22-d) \cdot (14+d) \cdot (d-6)}$$

4
4
14

$$p = \frac{a+b+c+d}{2} = 14$$

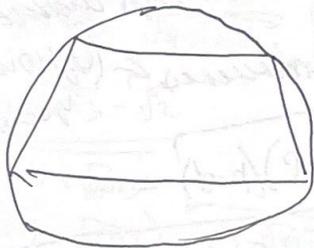
$$p = \frac{a+b+c+d}{2}$$

$$a+b+c-d$$



$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$$

$$= \frac{1}{4} \sqrt{\frac{b+c+d-a}{2}}$$

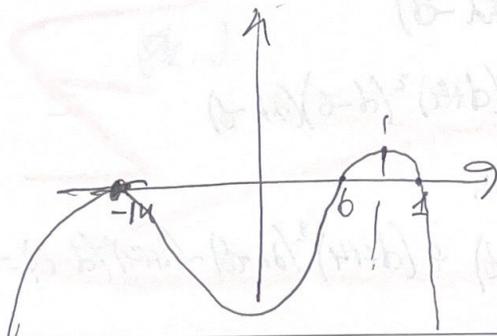


$$= \frac{1}{4} \sqrt{(d+\frac{d}{2}) \cdot (p+\frac{d}{2}) \cdot (d+\frac{d}{2}) \cdot (\psi-\frac{d}{2})}$$

$$\frac{1}{4} \sqrt{(d+14)^2 \cdot (d-6) \cdot (11-d)}$$

$$\frac{1}{4} \sqrt{14^2 \cdot (-6) \cdot (\psi-d)}$$

$$= \frac{1}{4} \sqrt{14 \cdot (11-d)}$$



Условие

№.

Всего 6 отрезков

~~Всего~~ Всего $C_6^2 = 15$ пар.

Тогда всего способов C_{15}^9 т.к.

Каждому способу мы и укладываем стороны
составим карточку условия.

№4.

$$a = 4 = b$$

$$c = 14 \quad S \rightarrow \max$$

$$d = ?$$

Забь формула Брахмагупты (или наоборот),

каждая масса, то $S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$ - $\frac{abcd}{\cos^2(\frac{D}{2})}$
 $d \rightarrow$ сумма противоположных углов.

\Rightarrow при заданных a, b, c масса d максимальна
 когда 4-х угловым вписанным (одно из необходимых
 условий).

$$\text{Тогда } S = \sqrt{(p-a)(p-b)(p-c)(p-d)} =$$

$$= \frac{1}{4} \sqrt{(b+c+d-a)(a-b+c+d)(a+b-c+d)(a+b+c-d)}$$

$$= \frac{1}{4} \sqrt{(d+14)^2(d-6)(22-d)}$$

Тогда пусть $f(d) = (d+14)^2(d-6)(22-d)$

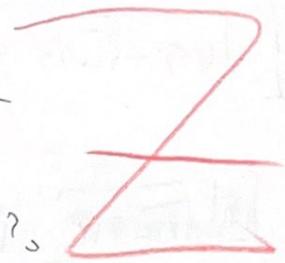
$S = \max$, при $f(d) = \max$.

$$f'(d) = 2(d+14) \cdot (d-6)(22-d) + (d+14)^2(22-d) - (d+14)^2(d-6) =$$

81-88-57-82
(89.13)

Черновик

$$\left[\sqrt{45 + \sqrt{2023}} - \sqrt{45 - \sqrt{2023}} \right] =$$



$$= \left[\frac{45 + \sqrt{2023} - 45 + \sqrt{2023}}{\sqrt{45 + \sqrt{2023}} + \sqrt{45 + \sqrt{2023}}} \right] =$$

45? $\leq 1600 + 400\sqrt{25}$
 $= 2000$

$$\left[\sqrt{\sqrt{2023} + 2} + \sqrt{2023} - \sqrt{\sqrt{2023} + 2} - \sqrt{2023} \right]$$

$$\sqrt{2025} - \sqrt{2023}$$

~~$\sqrt{45 + \sqrt{2023}} = a + b\sqrt{2}$~~

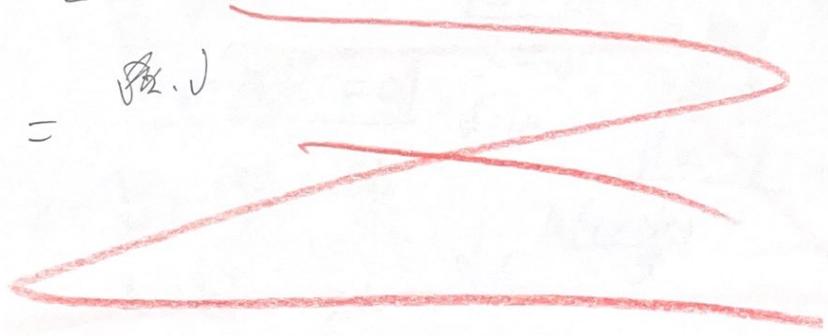
$$\sqrt{45 - \sqrt{2023}}$$

$$\left[\sqrt{45 + \sqrt{2023}} - \frac{\sqrt{2}}{\sqrt{45 + \sqrt{2023}}} \right] = \frac{\sqrt{2023} - \sqrt{45 + \sqrt{2023}} \cdot \sqrt{45 - \sqrt{2023}}}{\sqrt{45^2 - 2023}} = \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{45 + \sqrt{2023} - \sqrt{2}}{\sqrt{45 + \sqrt{2023}}}$$

$$\left[\frac{(\sqrt{45 + \sqrt{2023}})^2 \sqrt{45 - \sqrt{2023}} - (45 - \sqrt{2023}) \cdot \sqrt{45 + \sqrt{2023}}}{\sqrt{2}} \right]$$

$$= \sqrt{2} \cdot \sqrt{2}$$



Черновик.

$$\left[\sqrt{45 + \sqrt{2023}} - \sqrt{45 - \sqrt{2023}} \right]$$

$$\left[\sqrt{\sqrt{45 + \sqrt{2023}} - \sqrt{45 - \sqrt{2023}}} \right]$$

$$\sqrt{45 + \sqrt{2023}}$$

5

$$\left[\sqrt{45 + \sqrt{2023}} - \sqrt{45 - \sqrt{2023}} \right]$$

$$5 \frac{45 + \sqrt{2023} - \sqrt{2}}{\sqrt{45 + \sqrt{2023}}}$$

$$= \frac{\sqrt{2}}{45 + \sqrt{2023}} \sqrt{45 + \sqrt{2023}} - \frac{\sqrt{2}}{\sqrt{45 + \sqrt{2023}}}$$

$$\frac{\sqrt{2} - 45 + \sqrt{2023}}{\sqrt{45 + \sqrt{2023}}} 5 \frac{\sqrt{2}}{\sqrt{45 - \sqrt{2023}}} - \sqrt{45 + \sqrt{2023}}$$

81-88-57-82
(89.13)

Число

$|d| = \dots$
 \exists решение $[-\pi; \pi]$

$1234 \sin^2(x + \frac{\pi}{3}) - 789 \cos^2(2x + \frac{\pi}{3}) \leq 2023$

$\leq 1234 + 789 = 2023$

$\sin^2(x + \frac{\pi}{3}) = 1$
 $\cos^2(2x + \frac{\pi}{3}) = -1$

$\frac{1234}{+ 789}$
 $\underline{2023}$

$\sin(x + \frac{\pi}{3}) = \pm 1$
 $\cos(2x + \frac{\pi}{3}) = -1$

$x + \frac{\pi}{3} = \frac{\pi}{2} + \pi k$
 $x = \frac{\pi}{6} + \pi k$

$2x + \frac{\pi}{3} = \pi + 2\pi k$

$2x = \frac{5\pi}{3} + 2\pi k$

$d \neq 0$

$x = \frac{\pi}{6} + \frac{\pi k}{2}$

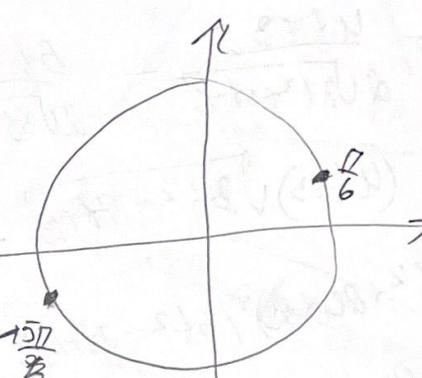
$x = \frac{2\pi}{3d} + \frac{2\pi k}{2} = \frac{\pi}{6}$

$d = 7k$

$4 + 12k = d$

$x = \frac{2\pi}{3d} + \frac{2\pi k}{2} = \frac{2\pi}{6}$

$4 + 12k = -5d$



$d = 4$

$d = 8$

$d = 16$

$d = 4$
 $d = 8$

$d = -\frac{4}{5}$

$d = \frac{4(1+3k)}{-5}$

Handwritten scribbles and notes on the right side of the page.

Черновик

$$\log_5 (x^2 - 9)^3 + 4 + \sqrt{3x^4 - 7x^2 + 19} = \sqrt{2x^4 + 3x^2 - 6}$$

$$x^2 = t$$

$$\log_5 (t - 9)^3 + 4 + \sqrt{3t^2 - 7t + 19} = \sqrt{2t^2 + 3t - 6}$$

$$f(t) = 3t^2 - 7t + 19$$

$$f_{\min} = f\left(\frac{7}{6}\right) = \frac{49}{6 \cdot 2} - \frac{7 \cdot 7}{6} + 19 = \frac{49}{6} \left(\frac{1}{2} - 1\right) + 19 = 19 - \frac{49}{6}$$

$$f(t) = \sqrt{2t^2 + 3t - 6} - \sqrt{3t^2 - 7t + 19}$$

$$f'(t) = \frac{4t+3}{2\sqrt{2t^2+3t-6}} - \frac{6t-7}{2\sqrt{3t^2-7t+19}}$$

$$(4t+3)\sqrt{3t^2-7t+19} = (6t-7)\sqrt{2t^2+3t-6}$$

$$(16t^2 + 24t + 9)(3t^2 - 7t + 19) = (36t^2 - 84t + 49)(2t^2 + 3t - 6)$$

$$48t^4 - 16 \cdot 7t^3 + 24 \cdot 3t^3 + 19 \cdot 16t^2 - 24 \cdot 7t^2 - 24 \cdot 7t^2 + 27 \cdot 2t^2 + 24 \cdot 9t - 7 \cdot 9t + 19 \cdot 9 =$$

$$24t^4 + 40t^3 - 60t^3 - 163t^2 - 396t^2 + 397t + 651 - 90 - 81 + 240 - 54 = 0$$

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(89.13)

$x^2 = 70$
Чернышев
 $\log_5 (|t-7|^3 + 1) + \sqrt{3t^2 - 7t + 19} = \sqrt{4t^2 + 3t - 6}$

$\frac{\sqrt{3 \cdot 29 - 35 + 19}}{\sqrt{59}}$

$\frac{\sqrt{4 \cdot 29 + 3 \cdot 29 - 6}}{\sqrt{59}}$

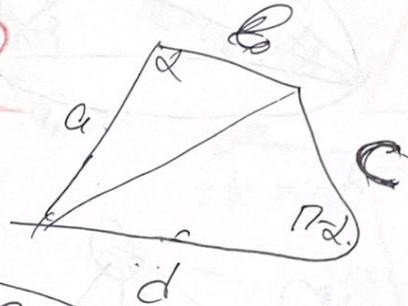
$t^2 - 10t + 25 = (t-5)^2$

$\log_5 (|t-7|^3 + 1) = \sqrt{4t^2 + 3t - 6} - \sqrt{(2t^2 + 3t - 6) + (t-7)^2}$
 $t = 5$

4 4 14

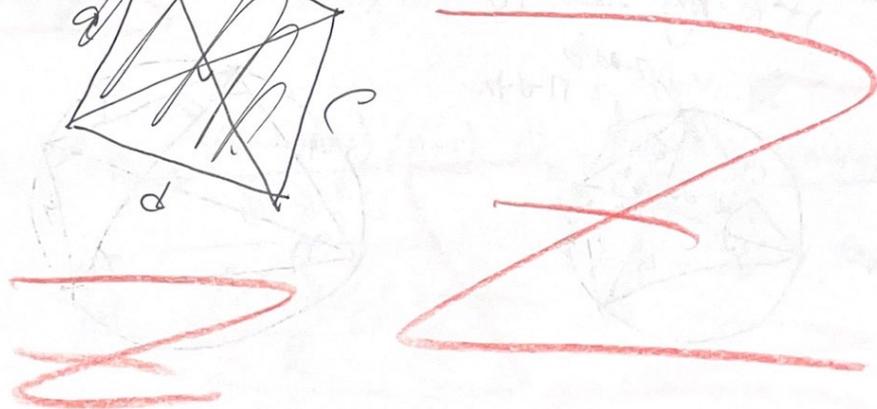
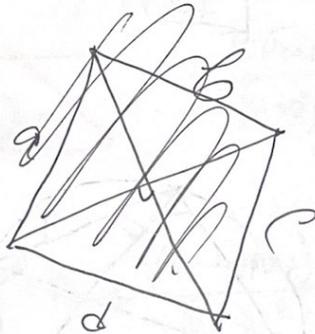
$S = \text{area}$

$d = ?$



$S = \sqrt{(p-a)(p-b)(p-c)(p-d)} = ab \cdot \cos \frac{\alpha + \beta}{2}$

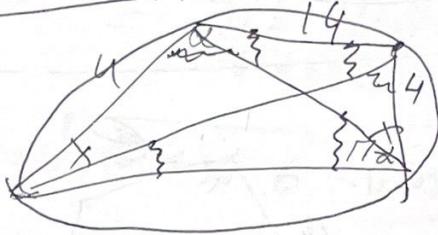
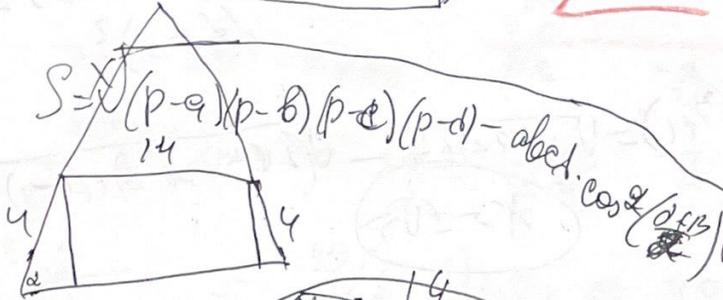
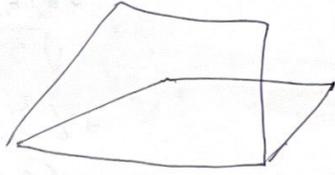
$S = d_1 \cdot d_2 \cdot \sin \alpha$, $\cos \alpha = \frac{a^2 + b^2 - c^2}{2ab}$



4; 4; 14. $S = \max$.
 $d = ?$

Черта

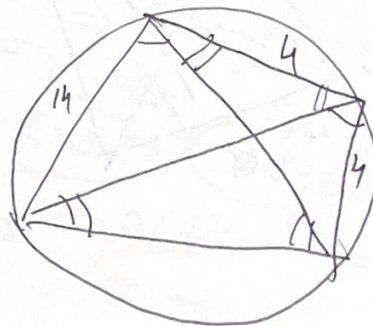
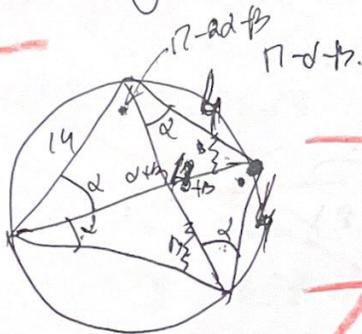
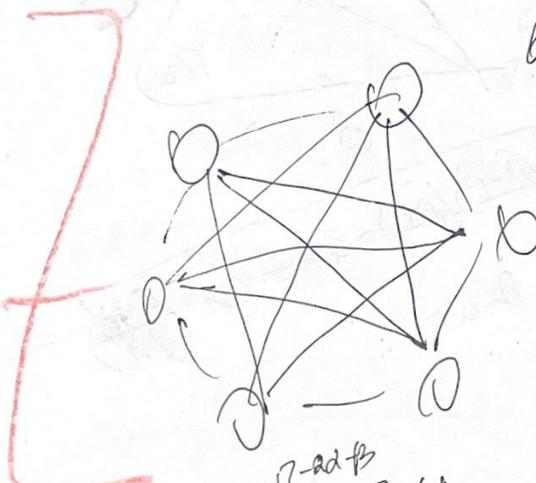
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2	2
4	4
8	8
16	7
32	5
64	10
128	11
256	13
512	8
1024	7

6 верши ребро = пересек

$$\frac{6 \cdot 7}{2} = 15$$



$$\sum_{k=0}^n C_k = 2^n$$

Цифровик

$$S(2^{2021}) + S(5^{2021}) = ?$$

$$a_1 = 13$$

$$a_n = \frac{(n^2+1) \cdot n}{(n-1)^2+1} \cdot a_{n-1}$$

$$a_{2022} = ?$$

$$a_{n-1} = a_{n-2} \cdot \frac{(n-1)^2+1}{(n-2)^2+1} \cdot (n-1)$$

$$= \frac{(n-1)^2+1}{(n-2)^2+1} \cdot \frac{n(n^2+1)}{(n-1)^2+1} \cdot a_{n-2}$$

$$\Rightarrow \frac{n(n-1)(n^2+1)}{(n-2)^2+1} a_{n-2} =$$

$$= \frac{n(n-1) \cdot (n^2+1)}{(n-2)^2+1} \cdot \frac{(n-2)}{(n-3)^2+1} \cdot a_{n-3} \cdot \frac{(n^2+1)}{(n-3)^2+1} \cdot a_{n-3} \dots$$

$$a_{n-2} \leq a_{n-3} \cdot \frac{(n-2)^2+1}{(n-3)^2+1} \cdot (n-2) \cdot a_n = \frac{n(n-1)(n-2) \dots (n-k+1) \cdot (n^2+1)}{(n-k)^2+1} \cdot a_{n-k}$$

$$a_n = \frac{(n^2+1) \cdot n!}{2} a_1$$

n	S(n)
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128
9	256
10	512
11	1024
12	2048
13	4096
14	8192
15	16384
16	32768
17	65536
18	131072
19	262144
20	524288
21	1048576
22	2097152
23	4194304
24	8388608
25	16777216
26	33554432
27	67108864
28	134217728
29	268435456
30	536870912
31	1073741824
32	2147483648
33	4294967296
34	8589934592
35	17179869184
36	34359738368
37	68719476736
38	137438953472
39	274877906944
40	549755813888
41	1099511627776
42	2199023255552
43	4398046511104
44	8796093022208
45	17592186044416
46	35184372088832
47	70368744177664
48	140737488355328
49	281474976710656
50	562949953421312
51	1125899906842624
52	2251799813685248
53	4503599627370496
54	9007199254740992
55	18014398509481984
56	36028797018963968
57	72057594037927936
58	144115188075855872
59	288230376151711744
60	576460752303423488
61	1152921504606846976
62	2305843009213693952
63	4611686018427387904
64	9223372036854775808
65	18446744073709551616
66	36893488147419103232
67	73786976294838206464
68	147573952589676412928
69	295147905179352825856
70	590295810358705651712
71	1180591620717411303424
72	2361183241434822606848
73	4722366482869645213696
74	9444732965739290427392
75	18889465931478580854784
76	37778931862957161709568
77	75557863725914323419136
78	151115727451828646838272
79	302231454903657293676544
80	604462909807314587353088
81	1208925819614629174706176
82	2417851639229258349412352
83	4835703278458516698824704
84	9671406556917033397649408
85	19342813113834066795298816
86	38685626227668133590597632
87	77371252455336267181195264
88	154742504910672534362390528
89	309485009821345068724781056
90	618970019642690137449562112
91	1237940039285380274899124224
92	2475880078570760549798248448
93	4951760157141521099596496896
94	9903520314283042199192993792
95	19807040628566084398385987584
96	39614081257132168796771975168
97	79228162514264337593543950336
98	158456325028528675187087900672
99	316912650057057350374175801344
100	633825300114114700748351602688

$$Q_n = \frac{(n^2+1)n!}{2} \cdot Q_1$$

Черновик.

$$n=1: Q_1 = Q_1 \oplus$$

$$Q_k = \frac{(k^2+1)k!}{2} \cdot Q_1$$

$$\frac{Q_{k+1}}{Q_k} = \frac{((k+1)^2+1) \cdot (k+1)!}{k^2+1}$$

$$Q_{k+1} = \frac{((k+1)^2+1)(k+1)!}{k^2+1} \cdot \frac{(k+1) \cdot k!}{2} =$$

$$= \frac{((k+1)^2+1) \cdot (k+1)!}{2} \oplus$$

$$Q_n = \sum_{i=1}^{n-1} \frac{(i^2+1)i!}{2} =$$

~~$$Q_n = \sum_{i=1}^{n-1} \frac{(i^2+1)i!}{2} =$$~~

$$Q_n = Q_{n-1} \cdot \frac{(n^2+1)n}{(n-1)^2+1}$$

$$Q_n + Q_{n-1} = Q_{n-1} \left(\frac{n^3+n+(n-1)^2+1}{(n-1)^2+1} \right) =$$

$$= Q_{n-1} \left(\frac{n^3+n^2-n+1}{(n-1)^2+1} \right)$$

$n^2 - 2n + 2$

$$Q_{n-1} = \frac{(n-1)^2+1}{(n-2)^2+1} \cdot Q_{n-2}$$

$$Q_n + Q_{n-1} + Q_{n-2} =$$

$$a_n = \frac{(n^2+1)n!}{2} a_1$$

$$\frac{a_{2022}}{\sum_{i=1}^{2022} a_i} = ?$$

Черновик

$$a_2 = \frac{5 \cdot 2}{2} \cdot a_1 = 5a_1$$

$$a_3 = 5 \cdot 6 \cdot a_1 = 30a_1$$

$$a_4 = 17 \cdot 12 a_1$$

a_1

$6a_1$

$36a_1$

$270a_1$

4.5.2

$$a_n = \sum_{k=1}^{n-1} \frac{(k^2+1)k!}{2}$$

$$\frac{17}{2} \cdot \frac{4!}{2} = \frac{17}{2} \cdot 12 = 102$$

$(17 \cdot 12) =$

$104 + 36 =$

$$\frac{a_n}{a_{n-1}} = \frac{(n^2+1)n}{(n-1)^2+1} = \frac{n^2+1}{n^2-2n+1} \cdot n$$

$$= \frac{(n^2-2n+2+2n-1)n}{n^2-2n+1} = 1 + \frac{(2n-1)n}{n^2-2n+1}$$

$$\frac{n^3+n}{n^2-2n+1}$$

$$\begin{array}{r} n^3 + 0n^2 + n + 0 \quad | \quad n^2 - 2n + 1 \\ \underline{n^3 - 2n^2 + n} \\ -dn^2 + 2n + 0 \\ \underline{-dn^2 + 4n - 4} \\ -2n + 4 \end{array}$$

$$\frac{n^2 + (-2n + 2n + 1) - 1}{(n-1)^2 + 1} =$$

$$= \frac{n^2 - 2n + 2 + 2n - 1}{(n-1)^2 + 1} = 1 + \frac{2n-1}{(n-1)^2 + 1}$$

$$a_n = \frac{(n^2+1)n}{(n-1)^2+1} a_{n-1}$$

$$a_{n-1} = \frac{(n-1)^2+1}{(n-1)^2+1} a_{n-1}$$

$$\left[\sqrt{45 + \sqrt{2023}} - \sqrt{45 - \sqrt{2023}} \right], \textcircled{1}$$

Чертовски

$$45 + \sqrt{2023} = a^2 + b^2 + 2ab \quad \textcircled{2} \quad \left[\sqrt{\frac{45 + \sqrt{2023}}{2}} - \sqrt{\frac{45 - \sqrt{2023}}{2}} \right]$$

$$a^2 + b^2 = 45$$

$$\sqrt{2023} = 2ab$$

$$a = \frac{\sqrt{2023}}{2b}$$

$$\textcircled{3} \quad \left[\sqrt{2} \right] \textcircled{1}$$

$$\textcircled{4} \quad \frac{2023}{4b^2} + b^2 = 45$$

$$2023 + 4b^4 = 180b^2$$

$$4b^4 - 180b^2 + 2023 = 0$$

$$D_1 = 90^2 - 4 \cdot 2023 = 8100 - 8092 = 8$$

$$b = \frac{90 \pm \sqrt{8}}{4}$$

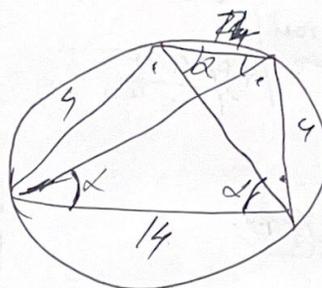
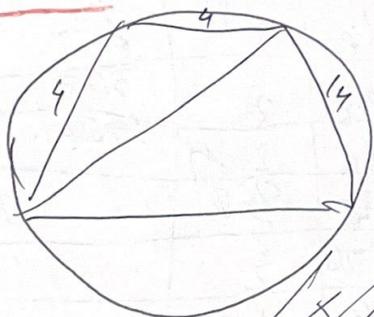
$$= \frac{45 \pm \sqrt{2}}{2} \quad \left(\sqrt{\frac{45 + \sqrt{2}}{2}} + \sqrt{\frac{45 - \sqrt{2}}{2}} \right)^2$$

$$= 45 + 2 \cdot \sqrt{(45 - \sqrt{2})(45 + \sqrt{2})}$$

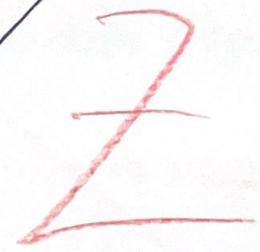
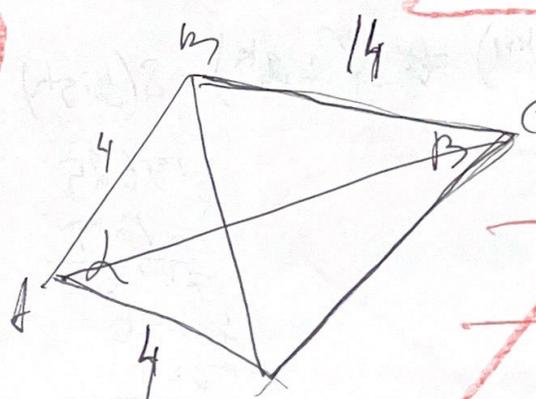
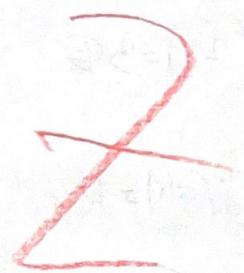
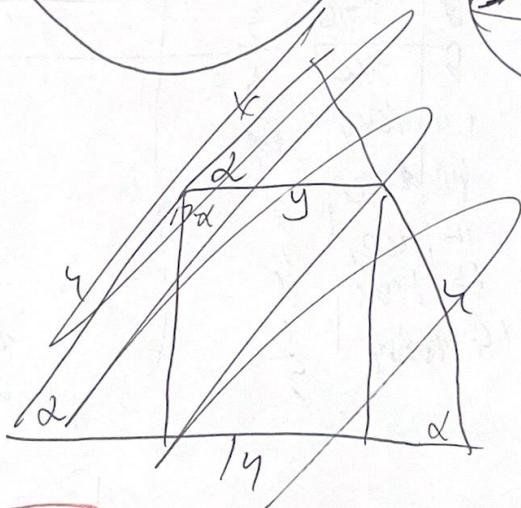
$$= 45 + \sqrt{45^2 - 2}$$

Шерман

4; 4; 14.



$$\frac{y}{14} = \frac{x}{4+x}$$



$$S = \frac{1}{2} \alpha \beta \sin \alpha$$

$$S = \frac{1}{2} (\overline{AB} \cdot \overline{AD} \sin \alpha + \overline{BC} \cdot \overline{CD} \sin \beta)$$

$S(n)$ - число цифр.

$S(2^{2021}) + S(5^{2021})$

$S(2^{2021}) \in S\left(\frac{10^{2021}}{2}\right)_5$

$\Rightarrow S\left(\frac{2^{2021}}{2}\right) + S\left(\frac{10^{2021}}{2}\right)$

~~2021~~

к 712

$S(2^k) + S(5^k) = k + 1$

$S(4) + S(25) = 3 + 2$

$S(2^k) + S(5^k) = k + 1$

$S(2^{k+1}) + S(5^{k+1}) = S(2 \cdot 2^k) + S(5 \cdot 5^k)$

Школа № 1

n	2^n	$S(2^n)$	5^n	$S(5^n)$
0	1	1	1	1
1	2	1	5	2
2	4	1	25	3
3	8	2	125	4
4	16	2	625	5
5	32	3	3125	6
6	64	3	15625	7
7	128	4	78125	8
8	256	4	390625	9
9	512	5	1953125	10
10	1024	5	9765625	11
11	2048	6	48828125	12
12	4096	6	244140625	13
13	8192	7	1220703125	14
14	16384	7	6103515625	15

1 - куча
2 - берца.

1 }
2 } - куча
3 }
4 }

5 }
6 } - берца
7 }
8 }

15625
x 125

178725
31350
17625

1954125

умножим

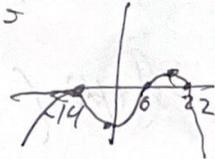
$$= (d+14)(2(d-6)(2d-6) + (d+14)(2d-6) + (d+14)(d-6)) +$$

$$> (d+14)(2(2ad-d^2-120-12+6d) + (d+14)(2d-6d)) +$$

$$> 2(d+14)(2ad-d^2-132 + (d+14)(14-d)) +$$

$$> 2(d+14)(-d^2+28d+132 - d^2+196) =$$

$$= 2(d+14)(-2d^2+47d+228) =$$



$$= 4(d+14)(-d^2+14d+114) = 4(d+14)(d-7)(d+17)$$

$$d_1 = 3$$

$$d_2 = 11$$

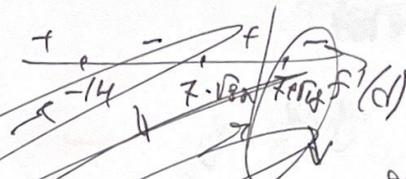
$$> -4(d+14)(d^2-14d-228) = -4(d+14)(d-(7+\sqrt{81}))(d-(7-\sqrt{81}))$$

$$D_1 = 49 + 81 = 130 \Rightarrow D_1 = 40 + 33 = 73$$

$$= 81$$

$$d = 14 \pm 9$$

$$d_{1,2} = 7 \pm \sqrt{81}$$



$$d = 7 \pm \sqrt{81}$$

На графике проверим, какие корни являются возможными. Проверим методом.

№6.

По индукции покажем

$$a_n = \frac{(n^2+1)n!}{2} a_1, \text{ при } n \geq 1, \text{ но что равно } a_1 \text{ нам не известно.}$$

Черновик

$$f(d) = (d-6) \cdot (d+4)^2 \cdot (d-6)/(1+d)$$

$$f'(d) = (d+4)^2 \cdot (1-6)$$

$$f'(d) = (d+4)^2(1+d) + 2(d+4)(d-6)(1+d) +$$

$$+ (d-6)(d+4)^2 =$$

$$= (d+4)^2$$

решить

$$S(2^{2021}) + S(5^{2021}) = \lceil 2021 \cdot \log_{10} 2 \rceil + \lceil 2021 \cdot \log_{10} 5 \rceil$$

$$= \lceil 2021 \cdot \log_{10} 2 \rceil + \lceil 2021 \cdot \log_{10} 5 \rceil$$

$$= \lceil \frac{2021}{\log_{10} 2} \rceil + \lceil \frac{2021}{\log_{10} 5} \rceil$$

10^k k+1 знаков

$$S\left(\frac{10^{2021}}{5^{2021}}\right) = S(2^{2021})$$

$a-1 < [a] \leq [a] \leq a$

$a-1 < [a] \leq a$

$$S\left(\frac{10^{2021}}{5^{2021}}\right) + S\left(\frac{10^{2021}}{2^{2021}}\right) = \lceil \frac{2021}{1+\log_2 5} \rceil + \lceil \frac{2021 \cdot \log_2 5}{1+\log_2 5} \rceil$$

$$S\left(\frac{10^{2021}}{5^{2021}}\right) = S(2^{2021})$$

$$S\left(\frac{10^{2021}}{2^{2021}}\right) + S(2^{2021})$$

$10^{2021} = 2^{2021} \cdot 5^{2021}$
k+1 знаков

$10^k = 2^k \cdot 5^k$

$$2^{2021} \cdot 5^{2021} = 2^{2021} + \frac{10^{2021}}{2^{2021}}$$

$$\Rightarrow \frac{4^{2021} - 10^{2021}}{2^{2021}}$$

$$S(2^{2021}) - S(5^{2021}) = S(10^{2021}) = 2^{022}$$

$$\frac{10^{2021}}{5^{2021}}$$

$$\frac{1}{2^{2021}}$$

$$S(2^{2021} \cdot 5^{2021}) = S(2^{2021}) + S(5^{2021})$$

$$2^{2021} = 1 + 1 + 1 + 1 + \dots + 1 \text{ (2021 times)}$$

$p + t = pt$
 $p \neq p$
 $pt - p - t = 0$
 $p(t-1) - (t-1) - 1 = 0$

$$\left(\frac{S(2^{2021})}{2^{2021}} - 1\right) \left(\frac{S(5^{2021})}{5^{2021}} - 1\right) = \dots$$

Упробову?

$$\textcircled{1} \lceil \log_{10}(2^{2011}) \rceil + \lceil \log_{10}(5^{2011}) \rceil >$$

$$> 2011$$

$$(n+1)^2 - n^2 = 2n+1$$

$$\lceil \log_{10}(2^{2011}) \rceil > \log_{10}(2^{2011})$$

1,3

$$\begin{aligned} 12 &= 120 + 34 > \\ &= 204. \end{aligned}$$

$\textcircled{1}$ $\textcircled{2}$

$$a \leq \lceil a \rceil \leq a+1$$

$$13 \times 12 = 120 + 26$$

$$a \in \mathbb{Z} \Rightarrow \lceil a \rceil \in \mathbb{Z}$$

$$\textcircled{3} \lceil \log_{10}(2^{2011}) \rceil + \lceil \log_{10}(5^{2011}) \rceil >$$

$$> 2011$$

$\textcircled{1}$ $\textcircled{2}$

$$2011 + 9 > 2013$$

(2011)

$Q_1 = 13$

Q_1

$$Q_2 = 5Q_1$$

$$\begin{aligned} 19 \\ 25 \\ 69 = 6 \cdot 11 \end{aligned}$$

$$Q_3 = 30Q_1$$

$$369 = 6 \cdot 61$$

$$Q_4 = 17 \cdot 12 =$$

$$2409 = 6 \cdot 401$$

$$Q_5 = 13 \cdot 120 =$$

$$1560 \quad 18009 = 6 \cdot 3001$$

$$Q_1 = 13$$

$$\frac{Q_n}{Q_{n-1}} = \frac{(n^2+1)}{(n-1)^2+1} \cdot n$$

$$Q_n = \frac{(n^2+1) \cdot n!}{2} Q_1$$

2

$$\frac{n! \cdot (n^2+1)}{2} + \frac{(n-1)! \cdot ((n-1)^2+1)}{2} + \frac{(n-2)! \cdot ((n-2)^2+1)}{2} + \dots$$

$$Q_n = Q_{n-1} \cdot \frac{(n^2+1)n}{(n-1)^2+1}$$

$$Q_n + Q_{n-1} = Q_{n-1} \left(\frac{(n^2+1)n + (n-1)^2+1}{(n-1)^2+1} \right) = Q_{n-1} \left(\frac{n(n^2+n-1) + (n-1)^2+1}{(n-1)^2+1} \right)$$

№3.

Шмелов

$$\log_5 (x^2 - 5|3+1) + \sqrt{3x^4 - 7x^2 + 19} = \sqrt{2x^4 + 3x^2 - 6}$$

$$\leq \log_5 (x^2 - 5|3+1) = \sqrt{2x^4 + 3x^2 - 6} - \sqrt{3x^4 - 7x^2 + 19} \leq 0$$

$$(x^2 - 5)^3 \geq 0$$

$$(x^2 - 5|3+1) \geq 2$$

$$\log_5 (x^2 - 5|3+1) \geq 0$$

$$\sqrt{2x^4 + 3x^2 - 6} - \sqrt{3x^4 - 7x^2 + 19} = \sqrt{2x^4 + 3x^2 - 6} - \sqrt{(2x^4 + 3x^2 - 6) + (x^2 - 5)^2}$$

≤ 0
на ОДЗ.

равенства достигаются при $x^2 = 5$.

Проверим также, что $2x^4 + 3x^2 - 6 \geq 0$ при $x^2 = 5$.

$$\Rightarrow x^2 \geq 2 \Rightarrow x = \pm \sqrt{5}$$

~~7~~ Ответ: $x = \pm \sqrt{5}$.

№5.

$S(n)$ — число цифр.

$$S(2^{2011}) \in S(5^{2011})$$

$S(n) = \lfloor \log_{10} n \rfloor + 1$, при $n \in \mathbb{N}$. И не является степенью 10-ки, при $n \neq 10^k$.

$$S(2^{2011}) \in S(5^{2011}) \Rightarrow \lfloor \log_{10} 2^{2011} \rfloor + 1 = \lfloor \log_{10} 5^{2011} \rfloor + 1 = A$$

$n \leq \lfloor n \rfloor + 1$
равенство, если $n \in \mathbb{N}$.

числовит

~~$\log_{10} 2^{2021}$~~

$$\log_{10} 2^{2021} < \lceil \log_{10} 2^{2021} \rceil < \log_{10} 2^{2021} + 1$$

т.к. $2^{2021} \neq 10^k$

~~7~~

$$\log_{10} 5^{2021} < \lceil \log_{10} 5^{2021} \rceil < \log_{10} 5^{2021} + 1$$

$$\log_{10} 2^{2021} + \log_{10} 5^{2021} < A < 2 + \log_{10} 2^{2021} + \log_{10} 5^{2021}$$

$$2021 < A < 2023$$

$$A \in \mathbb{N} \cap (2021, 2023) \Rightarrow \boxed{A = 2022}$$

Ответ: 2022.

~~7~~