



01-33-00-80  
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# МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ имени М.В.ЛОМОНОСОВА

Вариант 10 класс

Место проведения Краснодар  
город

## ПИСЬМЕННАЯ РАБОТА

Олимпиада школьников Ломоносов  
наименование олимпиады

по математике  
профиль олимпиады

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фамилия, имя, отчество участника (в родительном падеже)

Дата  
«29» марта 2026 года

Подпись участника  
И.Тужуева

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Чистовик  
Задача 4

50 (пятьдесят) *Минус*

$$\frac{a^{2x} - 3a^{x+1} + 2a^2}{\log_2 a} \geq 0$$

ОДЗ:  $\log_2 a \neq 0$   $a > 0$   
 $a \neq 1$

1 шаг.  $\log_2 a > 0 \Rightarrow a > 1$

$$\frac{a^{2x} - 3a^{x+1} + 2a^2}{\log_2 a} \geq 0 \quad | \cdot \log_2 a > 0$$

$$a^{2x} - 3a^{x+1} + 2a^2 \geq 0$$

$$a^2(a^{2x-2} - 3a^{x-1} + 2) \geq 0 \quad | : a^2 > 0$$

$$a^{2(x-1)} - 3a^{x-1} + 2 \geq 0 \quad a^{x-1} = t > 0$$

$$t^2 - 3t + 2 \geq 0$$

$$D = 9 - 8 = 1$$

$$t_1 = \frac{3+1}{2} = 2 = a^{x-1}$$

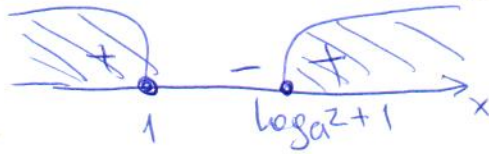
$$t_2 = 1 = a^{x-1}$$

$$x_2 - 1 = \log_a t = 0$$

$$x_2 = 1$$

$$\log_a 2 = x_1 - 1$$

$$x_1 = \log_a 2 + 1$$



будет  $\infty$  решений  $\Rightarrow$   
не подходит

2 шаг.  $\log_2 a < 0 \Rightarrow a \in (0; 1)$

$$\frac{a^{2x} - 3a^{x+1} + 2a^2}{\log_2 a} \geq 0 \quad | \cdot \log_2 a < 0$$

$$a^2(a^{2x-2} - 3a^{x-1} + 2) \leq 0 \quad | : a^2 > 0$$

$$a^{2(x-1)} - 3a^{x-1} + 2 \leq 0 \quad a^{x-1} = t > 0$$

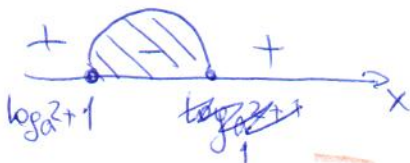
$$t^2 - 3t + 2 \leq 0$$

$$t_1 = 2 = a^{x-1}$$

$$x_1 = \log_a 2 + 1$$

$$t_2 = 1 = a^{x-1}$$

$$x_2 = 1$$



$$(\log_a 2 + 1) + (1) = 2026 \quad (\text{по условию})$$

$$-\log_a 2 = 2026$$

$$\log_a 2 = -2026$$

$$2 = a^{-2026}$$

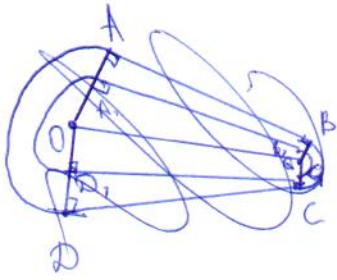
$$a = \sqrt[2026]{\frac{1}{2}} \quad (\text{т.к. } a > 0 \text{ корень } a = \frac{1}{\sqrt[2026]{2}} \text{ не подх.)}$$

Ответ:  $a = \sqrt[2026]{\frac{1}{2}}$



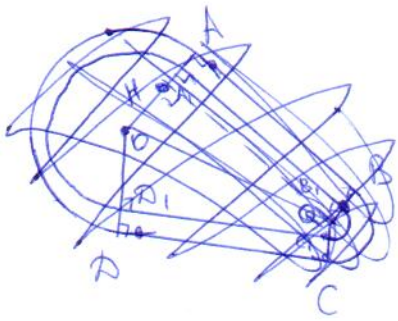
Чистовик Задача 7

Доказано:  $OA = OD = 4$ ;  $OQ = 6$ ;  $QB = QC = 1$ ;  $AA_1 = BB_1 = CC_1 = DD_1 =$



$OA \perp AB$  (т.к. касат.)  
 $QB \perp AB$   
 $AOA_1 \perp AA_1$   
 $QB_1 \perp AA_1$

1,5



1.  $AB$  - обш. касат.  $\Rightarrow$   
 $\Rightarrow AB = \sqrt{OQ^2 - (R-r)^2} = \sqrt{36 - (4-1)^2} =$   
 $= \sqrt{36 - 9} = 3\sqrt{3}$

2.  $\angle OAB = \angle QBA = \angle OAA_1 = \angle QBB_1 = 90^\circ \Rightarrow$   
 $\Rightarrow AA_1 = AB = 3\sqrt{3}$

3. Аналогично  $CC_1 = DD_1 = 3\sqrt{3}$

4. Проведем  $QH \perp OA \Rightarrow QH = 3\sqrt{3}$ ;  $\angle H = 90^\circ$

5.  $\angle H = 90^\circ$ ;  $OQ = 6$ ;  $OH = R - r = OA - QB = 4 - 1 = 3 \Rightarrow OH = \frac{1}{2}OQ \Rightarrow$   
 $\Rightarrow \angle OQH = 30^\circ \Rightarrow \angle HOQ = 60^\circ \Rightarrow \angle AOD = 120^\circ$

6.  $\angle OQH = 30^\circ \Rightarrow \angle OQB = 120^\circ \Rightarrow \angle CQB$  (больш.)  $= 240^\circ \Rightarrow$   
 $\Rightarrow \angle CQB$  (мал.)  $= 120^\circ$

7.  $OA_1 = OA - AA_1 = 4 - 1,5 = 2,5 = R_1$

~~8.  $\angle A_1OD_1 = 120^\circ \Rightarrow \angle A_1OD_1$  (больш.)  $= 240^\circ$~~   
 (мал.)

$\frac{l_{\text{полн.}}}{l_{A_1D_1, \text{б.}}} = \frac{360^\circ}{240^\circ} = \frac{3}{2} = \frac{2\pi R_1}{l_{A_1D_1, \text{б.}}} \Rightarrow$

$\Rightarrow l_{A_1D_1, \text{б.}} = 5\pi \cdot \frac{2}{3} = \frac{10\pi}{3}$

9.  $\angle B_1QC_1$  (мал.)  $= \angle B_1QB_1 - \angle C_1QB_1 = 1,5 - 1 = 0,5 = r_1$

10.  $\angle B_1QC_1$  (мал.)  $= 120^\circ \Rightarrow \frac{l_{\text{полн.}}}{l_{B_1C_1, \text{м.}}} = \frac{360^\circ}{120^\circ} = 3 = \frac{2\pi r_1}{l_{B_1C_1, \text{м.}}}$

$\Rightarrow l_{B_1C_1, \text{м.}} = \frac{\pi}{3}$

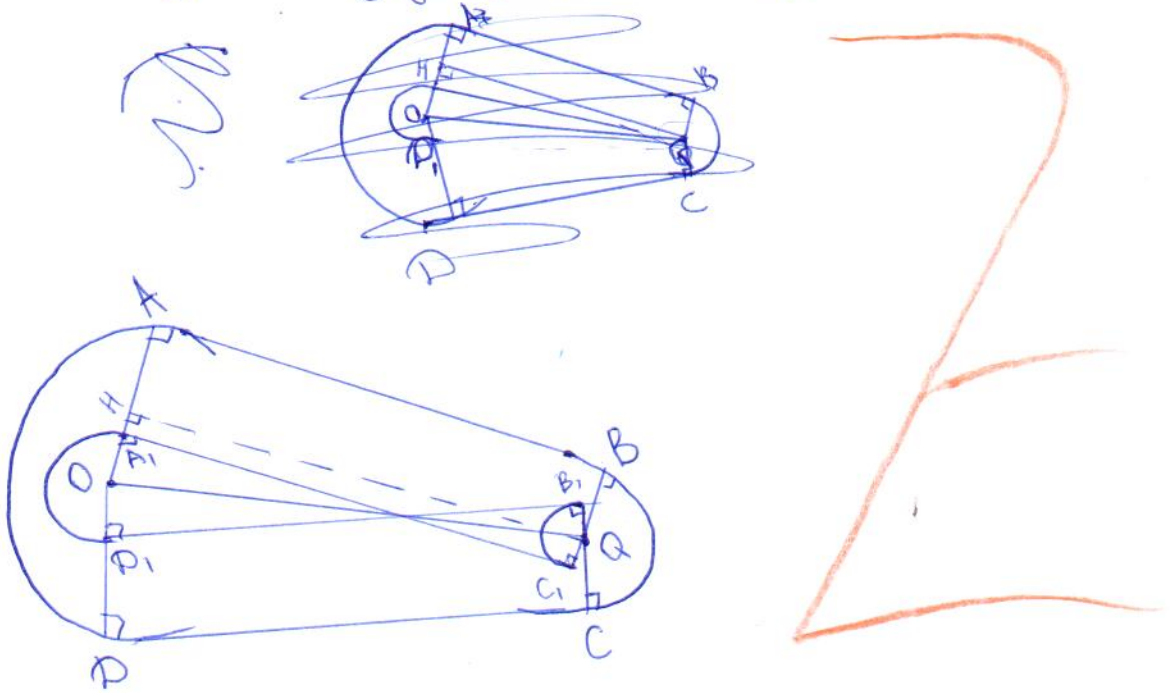
11. длина дорожки из травы  $= l_{A_1D_1, \text{б.}} + AA_1 + l_{B_1C_1, \text{м.}} + CC_1$

$= \frac{10\pi}{3} + 3\sqrt{3} + \frac{\pi}{3} + 3\sqrt{3} = \frac{11\pi}{3} + 6\sqrt{3}$

Ответ:  $\frac{11\pi}{3} + 6\sqrt{3}$

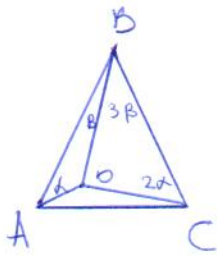
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Рисунок к задаче 7 Чистовик



Задача 6

Дано:  $\angle A = \angle C = 32^\circ$ ;  $\angle BAO = \alpha$ ;  $\angle BCO = 2\alpha$   
 $\angle ABO = \beta$ ;  $\angle CBO = 3\beta$



Найти:  $\frac{\angle BOA}{\angle BAO}$

1.  $\angle A = \angle C = 32^\circ \Rightarrow \angle ABC = 180^\circ - 32^\circ - 32^\circ = 116^\circ = 4\beta \Rightarrow \beta = 29^\circ$

2.  $\angle A = \angle C \Rightarrow AB = BC$

3.  $\angle BAO = \alpha$ ;  $\angle ABO = 29^\circ \Rightarrow \angle AOB = 180^\circ - (29^\circ + \alpha)$

$\angle OCB = 2\alpha$ ;  $\angle OBC = 87^\circ = 7\angle BOC = 180^\circ - (87^\circ + 2\alpha)$

4.  $\frac{OB}{\sin \alpha} = \frac{AB}{\sin \angle AOB} = \frac{AB}{\sin(180^\circ - (29^\circ + \alpha))} = \frac{AB}{\sin(29^\circ + \alpha)}$  (по т. синусов.)

$\frac{OB}{AB} = \frac{\sin \alpha}{\sin(29^\circ + \alpha)}$

$\frac{OB}{\sin 2\alpha} = \frac{BC}{\sin \angle BOC} = \frac{AB}{\sin(180^\circ - (87^\circ + 2\alpha))} = \frac{AB}{\sin(87^\circ + 2\alpha)}$  (по т. синусов.)

$\frac{OB}{AB} = \frac{\sin 2\alpha}{\sin(87^\circ + 2\alpha)} \Rightarrow \frac{\sin \alpha}{\sin(29^\circ + \alpha)} = \frac{\sin 2\alpha}{\sin(87^\circ + 2\alpha)}$

$\sin \alpha (\sin(87^\circ + 2\alpha)) = \sin 2\alpha \cdot \sin(29^\circ + \alpha)$

$\sin \alpha (\sin(87^\circ + 2\alpha)) = 2 \sin \alpha \cos \alpha \cdot \sin(29^\circ + \alpha)$

$\sin(87^\circ + 2\alpha) = 2 \cos \alpha \cdot \sin(29^\circ + \alpha)$

\* Чистовик

Задача 6 продолжение

$$\sin(87+2\alpha) = 2\cos\alpha \cdot \sin(29+\alpha) = \sin(29+2\alpha) + \sin(-29)$$

$$\sin 29^\circ = \sin(29+2\alpha) - \sin(87+2\alpha) = 2\cos(54+2\alpha)\sin(-29)$$

$$\sin 29^\circ = -\sin 29^\circ \cdot 2\cos(54+2\alpha)$$

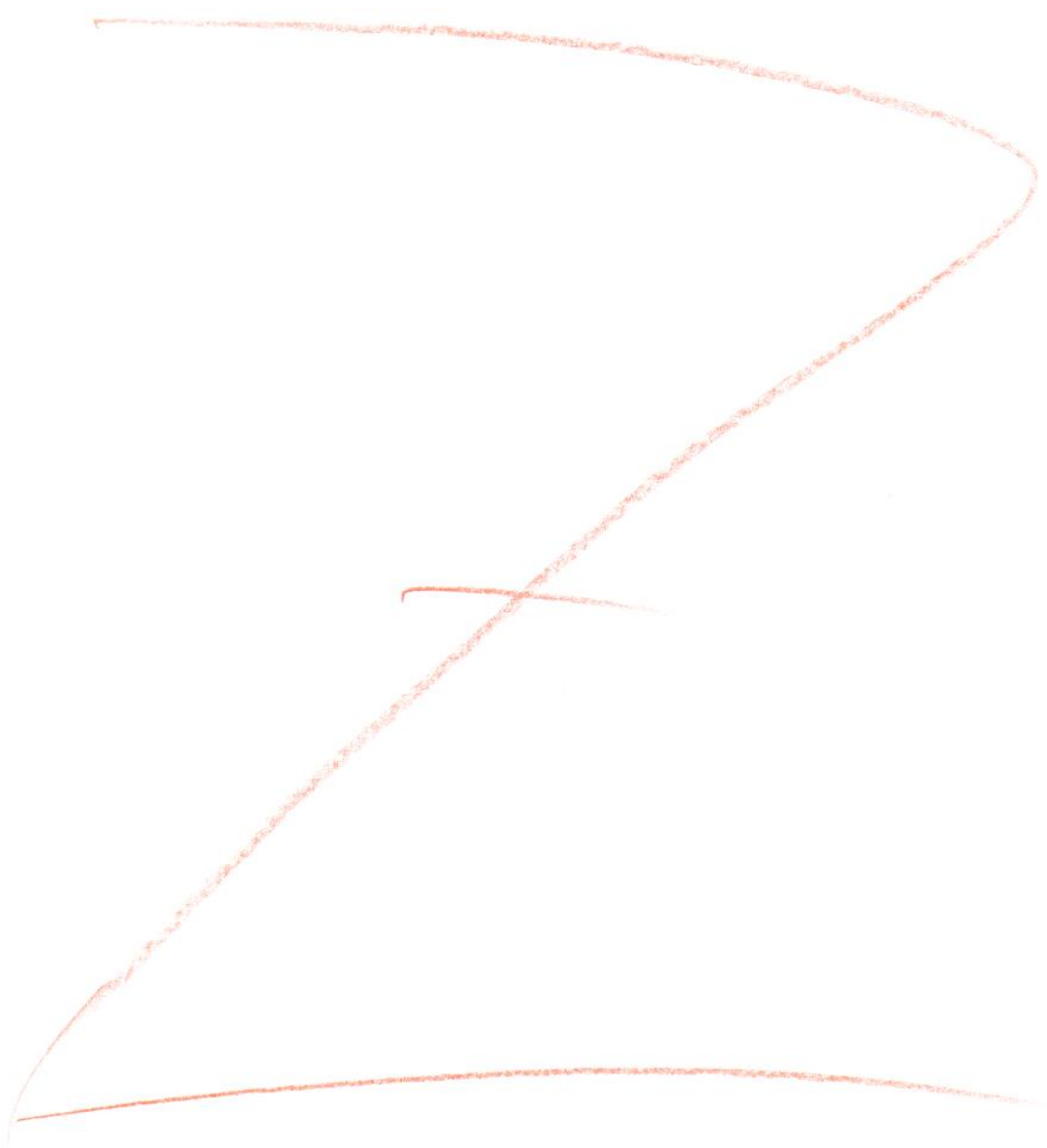
$$+2\cos(54+2\alpha) = 1$$

$$\cos(54+2\alpha) = +\frac{1}{2}$$

$$54+2\alpha = 60$$

$$2\alpha = 6$$

$$\alpha = 3 \Rightarrow \begin{matrix} \angle BOA = 148^\circ \\ \angle BAO = 3^\circ \end{matrix} \Leftrightarrow \frac{\angle BOA}{\angle BAO} = \frac{148}{3} = 49\frac{1}{3}$$

Ответ:  $49\frac{1}{3}$ 

Черновик

$$\frac{a^{2x} - 3a^x \cdot 3a + 2a^2}{\log_2 a} \geq 0$$

ОДЗ:  $\log_2 a \neq 0$   $a > 0$   
 $a \neq 1$

1 случай.  $\log_2 a > 0 \Rightarrow a > 1$

$$a^{2x} - 3a^x \cdot 3a + 2a^2 \geq 0$$

~~$a^x = t$~~

$$a^2(a^{2x-2} - 9a^{x-1} + 2) \geq 0 \quad | : a^2 > 0$$

$$a^{2(x-1)} - 9a^{x-1} + 2 \geq 0 \quad a^{x-1} = t$$

$$t^2 - 9t + 2 \geq 0$$

$$D = 81 - 8 = 73$$

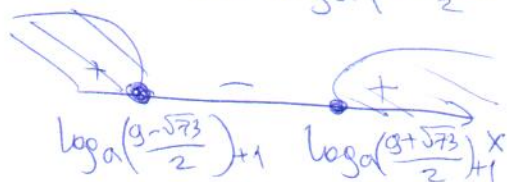
$$t_1 = \frac{9 + \sqrt{73}}{2} \quad t_2 = \frac{9 - \sqrt{73}}{2}$$

$$a^{x-1} = \frac{9 + \sqrt{73}}{2}$$

$$x_1 - 1 = \log_a \left( \frac{9 + \sqrt{73}}{2} \right)$$

$$x_1 = \log_a \left( \frac{9 + \sqrt{73}}{2} \right) + 1$$

$$x_2 = \log_a \left( \frac{9 - \sqrt{73}}{2} \right) + 1$$



Будет  $\infty$  реш.  $\Rightarrow$   
 $\Rightarrow$  не подходит

2 случай.  $\log_2 a < 0 \Rightarrow a \in (0, 1)$

$$a^{2x} - 9a \cdot a^x + 2a^2 \leq 0$$

$$a^2(a^{2(x-1)} - 9a^{x-1} + 2) \leq 0 \quad | : a^2 > 0$$

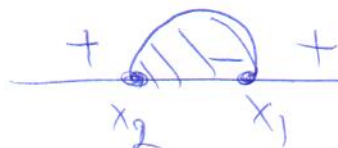
$$a^{2(x-1)} - 9a^{x-1} + 2 \leq 0 \quad a^{x-1} = t > 0$$

$$t^2 - 9t + 2 \leq 0$$

$$D = 81 - 8 = 73$$

$$x_1 = \log_a \left( \frac{9 + \sqrt{73}}{2} \right) + 1$$

$$x_2 = \log_a \left( \frac{9 - \sqrt{73}}{2} \right) + 1$$



$$x_1 - x_2 = 2026$$

$$\log_a \left( \frac{9 + \sqrt{73}}{2} \right) - \log_a \left( \frac{9 - \sqrt{73}}{2} \right) =$$

$$= \log_a \left( \frac{9 + \sqrt{73}}{2} \cdot \frac{2}{9 - \sqrt{73}} \right) = 2026$$

Героновик

$$\frac{OB}{\sin \alpha} = \frac{AB}{\sin(151-\alpha)} \Rightarrow \frac{OK}{AB} = \frac{\sin \alpha}{\sin(151-\alpha)} = \frac{\sin 2\alpha}{\sin(93-2\alpha)}$$

$$\frac{OB}{\sin 2\alpha} = \frac{AB}{\sin(93-2\alpha)}$$

$$abc + 2(ab+ac+bc) + 4a + 4b + 4c = 2028$$

$$abc + 2ab + 2ac + 2bc + 4a + 4b + 4c = 2026$$

$$abc + 2a(b+c) + 2bc + 4a + 4(b+c) = 2026$$

$$abc(a+2) + 2(b+c)(a+2) + 4a = 2026$$

$$(a+2)(bc + 2(b+c)) = 2026 - 4a$$

$$\left\{ \begin{aligned} bc + 2(b+c) &= 2026 - 4a \\ ac + 2(a+c) &= 2026 - 4b \\ ab + 2(a+b) &= 2026 - 4c \end{aligned} \right.$$

$$a = \frac{2026 - bc - 2b - 2c}{4}$$

$$b = \frac{2026 - ac - 2a - 2c}{4}$$

$$c = \frac{2026 - ab - 2a - 2b}{4}$$

$$\cos 2\alpha = 2\cos 54 \cdot \sin 2\alpha + \sin 2\alpha \cdot 2\sin 54 \cdot \sin 2\alpha = \sin 2\alpha \cdot \sin 54$$

$$2\cos 2\alpha \cdot \cos 54 + 2\sin 2\alpha \cdot \sin 54 = 1$$

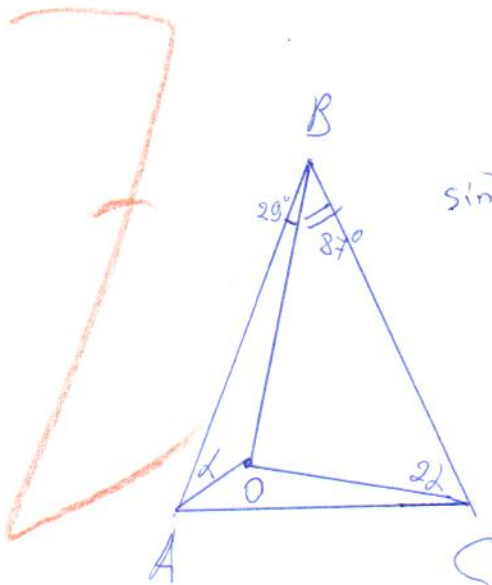
$$\cos(2\alpha - 54) = \frac{1}{2}$$

$$\cos(54 - 2\alpha) = \frac{1}{2}$$

$$\underline{54 - 2\alpha = 60}$$

$$\underline{2\alpha = 54 - 60}$$

Черновик



$\angle AOB = 151^\circ - \alpha$   
 $\angle AOC = 180^\circ - 64^\circ + 3\alpha$

~~$\sin^+ \cdot \cos^- + \cos^+ \cdot \sin^-$~~   
 ~~$\rightarrow -\sin^+ \cos^- \rightarrow \sin^- \cos^+$~~

$29 + 2\alpha - 87 + 2\alpha$   
 $\frac{-87}{-29}$   
 $\frac{58}{58}$

$180 - 54 - 2\alpha$

~~$\sin \alpha (\sin 93 \cdot \cos 93 - \cos 93 \sin 2\alpha)$~~   
 ~~$\sin \alpha \sin \beta = \sin(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}) \pm \sin(\frac{\beta+\alpha}{2} + \frac{\beta-\alpha}{2})$~~   
 ~~$= \sin(\frac{\alpha+\beta}{2}) \cos(\frac{\alpha-\beta}{2}) + \cos \frac{\alpha+\beta}{2} \sin \frac{\alpha-\beta}{2} \pm$~~

~~$+ \sin \frac{\alpha+\beta}{2} \cdot \cos \frac{\beta-\alpha}{2} + \cos \frac{\alpha+\beta}{2} \cdot \sin \frac{\beta-\alpha}{2} =$~~   
 ~~$= 2 \left( \frac{\cos \frac{\alpha+\beta}{2} \cdot \sin \frac{\alpha-\beta}{2} \right) =$~~

~~$\begin{cases} \frac{x+y}{2} = \alpha \\ x-y = 93-2\alpha \end{cases} \Rightarrow \begin{cases} x = 93-\alpha \\ x = 93-\alpha \end{cases}$~~   
 ~~$\begin{cases} \frac{x+y}{2} = \alpha \\ \frac{x-y}{2} = 29+2\alpha \end{cases} \Rightarrow \begin{cases} x = 29+2\alpha \\ y = \end{cases}$~~

~~$180 - 151 + 2\alpha =$~~   
 ~~$= 29 + 2\alpha$~~   
 ~~$87 + 2\alpha$~~

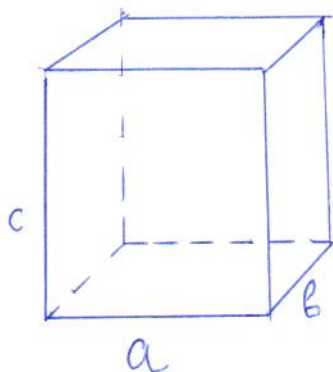
~~$\sin \alpha \cdot \sin(93 - 2\alpha) =$~~   
 ~~$\sin \alpha \cdot \sin(93 - 2\alpha) = \sin(151 - \alpha) \cdot \sin 2\alpha = \sin(151) \cdot$~~   
 ~~$\sin 2\alpha - \sin \alpha \cos 2\alpha$~~

~~$\frac{\sin(93 - 2\alpha)}{\sin 2\alpha} = 2 \sin(151 - \alpha) \cos \alpha$~~   
 ~~$= \sin 151 + \sin$~~

~~$\begin{cases} \frac{x+y}{2} = 151 - \alpha \\ \frac{x-y}{2} = \alpha \end{cases} \Rightarrow \begin{cases} x = 151 \\ y = 151 - 2\alpha \end{cases}$~~

~~$\sin(87 + 2\alpha) = \sin(29) \sin(29 + 2\alpha)$~~   
 ~~$\sin 87 \cdot \cos 2\alpha + \cos 87 \cdot \sin 2\alpha =$~~   
 ~~$= \sin 29 + \sin 29 \cdot \cos 2\alpha + \cos 29 \cdot \sin 2\alpha$~~   
 ~~$\cos 2\alpha (\sin 87 \sin 29) + \sin 2\alpha (\cos 87 - \cos 29)$~~   
 ~~$= \sin 29$~~

Гертвик



$$a \neq b \neq c; a, b, c \in \mathbb{N}$$

$$V_{\text{призмы}} = abc$$

$$S_{\text{пов}} = 2(ab + ac + bc)$$

$$abc + 2ab + 2ac + 2bc + 4a + 4b + 4c = 2026$$

abc мин - ?

$$ab(c+2) + 2a(c+2) + 2c(b+2) + 4b = 2026$$

$$a(c+2)(b+2) + 2c(b+2) = 2026 - 4b$$

$$(b+2)(ac + 2a + 2c) = 2026 - 4b$$

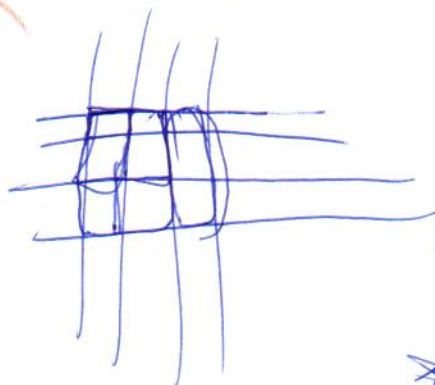
ac - zt

$$ac + 2(a+c) = \frac{2026 - 4b}{b+2}$$

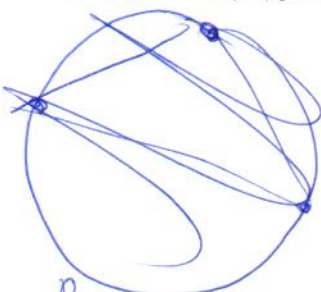
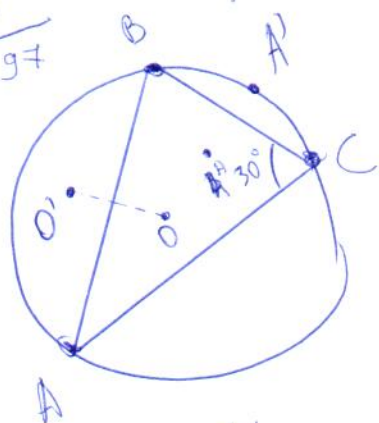
$$b^2 - 1 + 4b = 2026 - 4b$$

$$b^3 - b + 4b^2 + 2b^2 - 2 + 8b =$$

$$= b^3 + 6b^2 + 11b - 2028 = 0$$

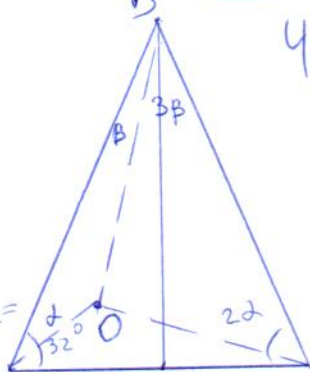


$$\begin{array}{r} 22 \\ 165 \\ \times 13 \\ \hline 4507 \\ 169 \\ \hline 2197 \end{array}$$



$$4\beta = 180 - 64$$

$$\beta = 45 - 16 = 29$$



$$\angle OAC = 32 - 2\alpha$$

$$\angle ACO = 32 - 2\alpha \Rightarrow$$

$$\rightarrow \angle AOC = 180 - 32 + 2\alpha - 32 + 2\alpha =$$

$$= 180 - 64 + 4\alpha = A$$

$$= \angle AOC = 116 + 4\alpha$$

$$\angle AOB = 180 - 2\alpha - \beta$$

$$\angle BOC = 180 - 3\beta - 2\alpha$$

$$180 - 29 - \alpha =$$

$$= \frac{151 - \alpha}{\alpha} - ?$$

$$\Rightarrow 116 + 4\alpha + 180 - \beta + 180 - 3\beta - 2\alpha = 360$$

$$116 = 4\beta \Rightarrow 2028 = 2^2 \cdot 507 =$$

$$= 2^2 \cdot 3 \cdot 169$$

$$\beta = 29$$